A STUDY OF RADAR CLUTTER MODELS

By

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INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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MASTER OΓ TECHNOLOGY

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CERTIFICATE

This i to certify that the thesis entitled A STUDY OF RADAR CLUTTER MODELS by Iandava Raj ari ha been carried out under my supervision and has not been ubmitted elsewher for a degree

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(MANDAVA RAJESWARI)

ABSTRACT

Clutter which degrades the probability of target detection performance of a radar is a term used to denote the composite echoes from unwanted targets illuminated by the radar beam Clutter return unlike additive thermal noise depend on radar s own transmission Therefore mere increase in the transmitted power without a suitably chosen signalling waveform does not necessarily result in improved target detection performance Considerations of optimum waveform and receiver designs presume a knowledge of the statistical description of clutter returns Specifications of the scattering function which provides a measure of the distribution of clutter power in delay and doppler variables is often considered adequate

This thesis deals with the evaluation of the scattering function for two models of clutter In the first the scatterers are modelled as random rotating dipoles with an overall drift velocity and differential velocities An expression for the scattering function is derived by calculating the voltage reflection coefficients of an ensemble of dipoles with non homogeneous Poisson distribution An attempt has been made to generate a few scattering functions which compare favourably with some of the reported experimental results

In the second approach clutter targets are modelled as a collection of ellipsoidal scatterers The ellipsoids are assumed to fluctuate in size and l o about their mean positions As an illustration an expression for the scattering function is obtained for a relatively simple geometry and aspect angle fluctuations

CONTENTS

CHAPTER I

INTRODUCTION

Clutter is a term used to denote the echoes from unwanted radar targets illuminated by the radar beam. Targets are classified as clutter in relation to the application of the radar. Unlike thermal noise, clutter returns are due to radar's own transmission. Therefore increasing the signal power alone without choosing a suitable transmitting waveform does not improve the target detection performance. For designing the transmitting waveform and the optimum radar receiver to operate effectively in the presence of clutter, it is necessary to develope a theoretical model of clutter.

The characteristic that is often choosen to describe the echo signal from extended clutter is the scattering cross section per unit intercepted area σ^o, which is independent of the size of the clutter patch illuminated. This parameter depends on the specific features of the clutter producing targets such as surface roughness, dielectric constant, number of scatterers and the transmitting frequency. But the static cross-section σ^o is inadequate to characterize the time varying statistical nature of the clutter returns from extended fluctuating targets. A second order description of the clutter return is provided by the scattering function associated with the target along with the envelope of the transmitted waveform

The scattering function characteri ation of clutter which provides a measure of the distribution of lutter pover in delay and doppler variables is often considered adequate for designing the transmitted wav form and the optimum r ceiver

1 1 DEFINITION OF THE SCATTERING FUNCTION

An extended target may be modelled as a random linear time-varying filter and the echo return from such targ t is then given as the output of the filt r whose input is the waveform transmitted by the radar The complex envelope of the clutter returns can be written as

$$S_r(t) = \int_{\infty}^{\infty} f(t \quad \lambda) b((t \quad \tfrac{\lambda}{2}) \quad \lambda)\, d\lambda \qquad (1\ 1)$$

where $\tilde{f}(t \quad \lambda)$ is the complex envelope of the transmitt d signal $b(t \quad \frac{\lambda}{2} \quad \lambda)\, d\lambda$ i the return from the targ t located within the range interval $(\lambda \quad \lambda + d\lambda)$ when an unit impul is transmitted at $(t \quad \lambda)$ nd λ is the round trip delay time The correlation function of the received signal ssuming $\tilde{S}_r(t)$ has zero mean is giv n by

$$K_{\tilde{S}_r}(t_\alpha \quad t_\beta) = E\{ \iint_{\infty}^{\infty} \tilde{f}(t_\alpha \quad \lambda)\, f^*(t_\beta \quad \lambda)\, b(t \quad \tfrac{\lambda}{2} \quad \lambda)$$
$$b^*(t - \tfrac{\lambda_1}{2} \quad \lambda_1)\, d\lambda \quad d\lambda_1 \} \qquad (1\ 2)$$

Assuming that the returns from different interval a statis-tically independent and the $b\ (t\, \lambda)$ is a tationary random proc s eqn (1 2) reduces to

$$K_{S_r}(t_\alpha\ t_\beta) = \int_{\infty}^{\infty} \tilde{f}(t_\alpha \quad \lambda)\, K_{DR}(t_\alpha - t_\beta \quad \lambda)\, \tilde{f}^*(t_\beta - \lambda)\, d\lambda \quad (1\ 3)$$

The scattering fun tion S_{DR} (f λ) is defined as

$$S_{DR}(f \quad \lambda) \triangleq \int_{\infty}^{\infty} K_{DR}(\tau \ \lambda) \ e^{-j2\pi f \tau} d\tau \qquad (1\ 4)$$

where $(t_\alpha \quad t_\beta) = \tau$

Hence

$$\langle_{S_r}(t_\alpha \quad t_\beta) - \iint_{\infty}^{\infty} f(t_\alpha \quad \lambda) \ S_{DR}(f \ \lambda) \ \tilde{f}^*(t_\beta \ \lambda) \ e^{+j2\pi f \tau} d\tau d\lambda \qquad (1\ 5)$$

It may be noted that

$$2 \ \sigma_b^2 \qquad \iint_{-\infty}^{\infty} S_{DR}(f \ \lambda) \ df \ d\lambda$$

repr sents the ratio of the xpected value of the re eived energy and includes in it the antenna gains p th lo e s and average radar cross ection of the clutter target

1 2 PAST RELATED WOR

In th past some theoreti al models which a count for the statistical natur of the clutter returns are propos d Siegert [1] Kerr [8] L son and Uhl nbeck [9] pre ented the first two pr bability distribution functions of th echo power elly and Lerner [14] d veloped a theoretical mod l of a haff cloud from which they derived various tati tical properties of the cho The signal return from the haff loud was considered as a random pro ess and the scatter rs wer treated a points with variabl cross sections Vantrees [3] in his model derived an expression for the correlation function of the clutter returns The returns from the scatterers in the

ıllumınated volume was as umed to constıtut a non homogeneous Foısson process The strength of the echoes and lela/s of the s atterers were con ıderel as random varıable

Chılders and R ed [4] consıdered the clutter cloud as collectıon of poınt scatterers movıng about and reflectıng en rgv ındependently of one another The effe t of scatterer rotatıon and multıple s atterın were ıgnored Ampl tude reflectıon coeffıcıent and the delay of the sc tterers were as umed to be random varıables As ın the prevıous model treatıng th returns from the catterers ın the ıllumınated volume as a non homogeneous Poısson process the tıme varyıng correlatıon functıon for a radar back scattered noıse pro ess wa determın d Subsequently an expressı n for the power spectrum wa obtaıned as umıng the clutter noıse proces to be statıonary

In the model formulated by Wong Reed and Kaprıelıan [1] ıt was assumed that clutter loud ı an ensemble of r ndom dıpoles havıng lınear and angular velocıtıe An expressıon for the tıme varyıng correlatıon functıon of the echo sıgnal was derıved ın terms of the characterıstıcs of th transmıtted waveform polarızatıon and the dıstrıbutıon of the scatterers An expressıon for the power spectrum was obtaıned wıth suıtable assumptıons

J W rıght [2] determıned the statıstıcal characterıstıcs of the scatterıng parameters such as RCS elevatıon error azımuth

error and the target phase of an air raft type target The target was d vided into a finite deterministic number of ellipsoids with varying r dar cro se tions The aspect angles of the ellipsoids were treat d as random proc ses to allow fluctuations of the target

1 3 ORGA I ATION OF THE THESIS

In this thesis two models of clutter are proposed and the scattering function is derived for each hapter 2 consi ts of the first model in which the clutter cloud is treated as random dipol s with linear and ngular velociti s The round trip delays are considered as random variables and the echo arrivals are assumed to constitute a non-homogeneous Poi on pro ess With th se assumptions an expression for the catt ring function in delay and doppler domain is obtained

The second model is given in Chapter 3 In this model the clutter cloud is considered as a collection of f nite deterministic number of ellipsoidal scatt rers An expression for the voltage reflection coefficient is derived followed by the derivation of the scattering function

The results of Chapter 2 and Chapter 3 are discussed upon and concluded in Chapter 4

CHAPTER II

ROTATING RANDOM DIPOLE SCATTERER MODEL

In this chapter an expression for the scattering function is derived where the clutter cloud is assumed to be a collection of random dipoles The dipoles are assumed to be random to represent the random motions of the objects such as leaves branches etc under the effects of wind forces

In Section 2 1 scattering function with delay and doppler variables is derived for a collection of random scatterers Section 2 2 consists of a detailed description of the model a derivation of the voltage reflection coefficient of a dipole and the derivation of the scattering function of the same model of clutter It also contains the evaluation of the scattering function where the random variables of interest are assumed to be Gaussian Some of the experimental results are simulated using the model of section 2 2 and the results of the simulation are reported in Section 2 3

2 1 SCATTERING FUNCTION DERIVATION

When a signal is transmitted by radar it is scattered by various objects in the illuminated volume which are called scatterers Therefore the returned signal at any time t is a sum of the echoes from a large number of scatterers If $\tilde{s}_t(t)$ is the complex envelope of the transmitted signal the

complex envelope of the returned signal $S_r(t)$ is given by

$$\tilde{S}_r(t) = \sum_{j-1}^{N_t} Z_j(t \quad \frac{\tau_j}{2}) \; S_t(t \quad \tau_j) \qquad (2\ 1\ 1)$$

where N_t is the number of catt rers which cause an echo to arrive in the interval (T T) and is assumed to constitute a non homog neous Poisson proce s with rate $\nu(t)$ Z_j is the strength of the echo of the jth scatterer and is assumed to be a stationary random process with zero mean The processes Z_i $\tilde{Z}_j$ for $i \neq j$ are assumed to be statistically independent Another assumption made here is that the round trip delays τ_j s are random variable τ_js can be thought of as the unordered delays of the scatterers which give an echo in the interval (T T) From the Poisson process a sumption given that K echoes have rrived in the interval (T T) the unordered delays τ_1 τ_2 τ_K are mutually independent Since N_t is a non homogeneous Poisson process they are assumed to hav a common density function of the form

$$f_{\tau_j}(\tau_j / N_t = K) = \frac{\nu(\tau_j)}{\int_{T}^{T} \nu(t)\, dt} \qquad (2\ 1\ 2)$$

Since the scattering function should be evaluated in delay and doppler domain it i necessary to calculate the auto correlation of the returned ignal where the returned signal is a sum of the echoes from all those scatterers which

contribute to the same delay i e the random variables τ_j $j = 1$ K should all take a single value say λ in the interv l (T T)

The probability that the random variable τ_j takes a value betwe n $(\lambda \ \lambda + d\lambda)$ wher $d\lambda \to o$ i

$$P_{\tau_j}[\lambda \leq \tau_j < \lambda \ d\lambda] = \frac{\nu(\lambda)\, d\lambda}{\int_T^T \nu(t)\, dt} \tag{2 1 3}$$

Autocorrelation of the r ceived signal is

$$R_{S_r}(t_\alpha \ t_\beta)\Big|_{\lambda < \tau_{j_{\forall j}} < \lambda + d\lambda} = E\{S_r(t_\alpha) \ \tilde{S}_r^*(t_\beta)\}\Big|_{\lambda < \tau_{j_{\forall j}} < \lambda + d\lambda} \tag{2 1 4}$$

$$= \sum_{k=o}^{\infty} E[\tilde{S}_r(t_\alpha) \ S^*(t_\beta)/N_t = K]\Big|_{\lambda < \tau_{j_{\forall j}} < \lambda + d\lambda} Pr[N_t = K] \tag{2 1 5}$$

$$E\{S_r(t_\alpha) \ \tilde{S}_r^*(t_\beta)/N_t \ K\}\Big|_{\lambda < \tau_{j_{\forall j}} \leq \lambda + d\lambda}$$

$$= E\{\sum_{j-1}^{K} \sum_{i=1}^{K} \tilde{Z}_j(t_\alpha - \frac{\tau_j}{2}) \ \tilde{Z}_i(t_\beta \ \frac{\tau_i}{2}) \ \tilde{S}_t(t_\alpha \ \tau_j) \ \tilde{S}_t^*(t_\beta \ \tau_i)/N_t - K\}\Big|_{\lambda < \tau_{j_{\forall j}} \leq \lambda + d\lambda} \tag{2 1 6}$$

Since $\tilde{Z}_j$s are independent ith zero mean $\forall$ j eqn (2 1 6) reduces to

$$E[\tilde{S}_r(t_\alpha)\ S_r^*(t_\beta)/N_t = K]\Big|_{\lambda < \tau_J < \lambda + d\lambda}$$

$$= \sum_{J=1}^{K} E\{[Z_J(t_\alpha \ \tfrac{\tau_J}{2})\ \tilde{Z}_J^*(t_\beta - \tfrac{\tau_J}{2}) S_t(t_\alpha \ \tau_J) S_t^*(t_\beta \ \tau_J)]\ /N_t = K\}\Big|_{\lambda \ \tau_J < \lambda + d\lambda} \qquad (2\ 1\ 7)$$

$$= \sum_{J=1}^{K} E[Z_J(t_\alpha \ \tfrac{\lambda}{2})\ {}_J^*(t_\beta \ \tfrac{\lambda}{2})]\ \frac{\nu(\lambda)\ d\lambda}{\int_T \nu(t)\ dt}\ S_t(t_\alpha - \lambda) S_t^*(t_\beta \ \lambda)$$

$$= K\ E\ \{Z_J(t_\alpha \ \tfrac{\lambda}{2})\ Z_J^*(t_\beta - \tfrac{\lambda}{2})\}\ \frac{\nu(\lambda)\ d\lambda}{\int_T \nu(t)\ dt}\ S_t(t_\alpha \ \lambda)\ \tilde{S}_t^*(t_\beta \ \lambda) \qquad (2\ 1\ 8)$$

where ${}_J$s are assumed to be identically distributed

Assuming the process Z_J to be stationary

$$E\{\ {}_r(t_\alpha)\ \tilde{S}_r^*(t_\beta)/N_t = K\}\ d\lambda = K\ R_Z(\tau)\ \frac{\nu(\lambda) d\lambda}{\int_T \nu(t)\ dt}\ \tilde{S}_t(t_\alpha - \lambda) S_t^*(t_\beta \ \lambda) \qquad (2\ 1\ 9)$$

Since

$$\underset{d\lambda \to 0}{Lt}\ E\{\tilde{S}_r(t_\alpha)\ S_r^*(t_\beta)/N_t = K\}\Big|_{\lambda < \tau_J < \lambda + d\lambda} = E\{\tilde{S}_r(t_\alpha)\ \tilde{S}_r^*(t_\beta)/N_t = K\}\ d\lambda \qquad (2\ 1\ 10)$$

$$R_{\tilde{r}}(t_\alpha \ t_\beta \ \lambda) = R_Z(\tau)\ \tilde{S}_t(t_\alpha \ \lambda) \tilde{S}_t^*(t_\beta - \lambda)\ \frac{\nu(\lambda)}{\int_{-T}^{T} \nu(t) dt}\ \sum_{K=0}^{\infty} I\ Pr[Nt\ K]$$

$$= R_{\tilde{Z}}(\tau)\ \tilde{S}_t(t_\alpha - \lambda)\ S_t^*(t_\beta \ \lambda)\ \nu(\lambda) \qquad (2\ 1\ 11)$$

$$\text{because} \sum_{K=0}^{\infty} K \Pr[N_t = K] = \int_{T}^{T} \nu(t)\, dt \qquad (2.1.12)$$

From (2 1 11) the auto or elation of the proce s $\tilde{}$ is R ($\hat{}$)

$$\text{Let} \quad R_{\tilde{y}}(\tau, \lambda) \triangleq R(\tau)\,\nu(\lambda) \qquad (2.1.13)$$

Then the normalized correlation function is

$$\varsigma(\tau, \lambda) = \frac{R_x(\tau, \lambda)}{R_x(0, \lambda)} \qquad (2.1.14)$$

Therefore the catterin fun tion is given by

$$(f, \lambda) = \int_{-\infty}^{\infty} g(\tau, \lambda)\, e^{-j2\pi f\tau}\, d\tau \qquad (2.1.15)$$

Random dipole model for radar clutter was initi lly uggested in [1] by J L ong I S Reed et l in which an expre sion for the fluctu tion frequency spectrum was derived For the same model in the present work a different and an easier approach is taken to calculate the voltage reflection coefficient an expre sion for the scattering function in delay and doppler domain is deriv d

In this model it is assumed that the illuminated volume con ist of a collection of random dipole scatterers refle cting en rgy independently the cloud of scatterers h an overall drift velocity and the scatterers have diff rential velocities In addition to these it is also ssumed that the scatterers have rotational motion about an axis perpendicular to the dipole axis For land clutter rotational motion or equivalent movement of branches leaves etc undcr the

effects of windforces is a major contributing factor to the fluctuation of echo intensity which can not be neglected The rate of echo arrival depend on the local density of the loud of scatterers

2 2 1 VOLTAGE REFLECTION COEFFICIENT

To calculate the voltage reflection coefficient for a dipole whose axis is oriented parallel to the unit vector d in the target fixed coordinate system as shown in Fig (1) it is assumed that the dipole is rotating about an axis S which is perpendicular to the axis of the dipole The axis S is defined by the angle ξ and η as shown Let (u v s) be a set of orthogonal axes such that the plane u v is the same as the plane of rotation of the dipole Therefore the transformation between the coordinate systems where η_o and ξ are right handed angles about z and the new x axis respectively is given by

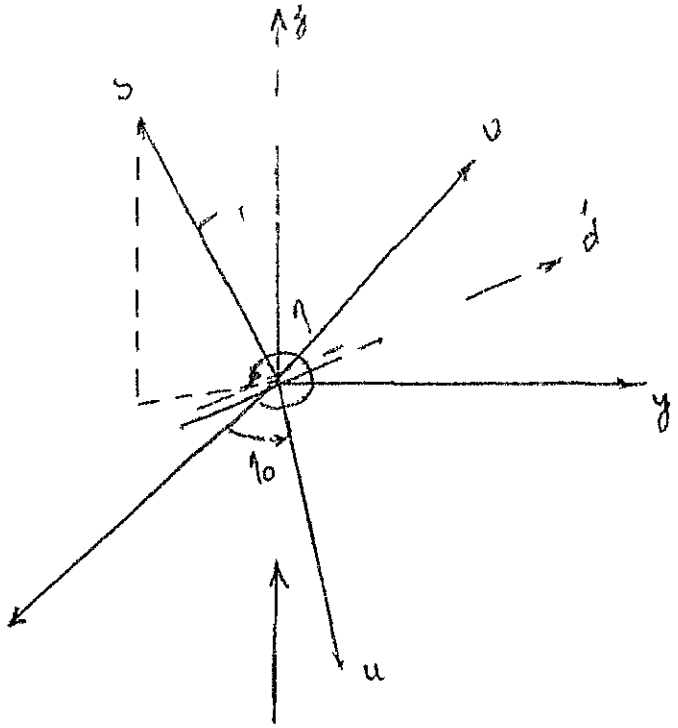
$$\begin{vmatrix} a_u \\ a_v \\ a_s \end{vmatrix} = \begin{bmatrix} \sin\eta & \cos\eta & 0 \\ \cos\xi \quad \cos\eta & \cos\xi \quad \sin\eta & \sin\xi \\ \sin\xi \quad \cos\eta & + \sin\xi \quad \sin\eta & \cos\xi \end{bmatrix}^{T} \begin{vmatrix} \hat{i}_t \\ \hat{j}_t \\ \hat{k}_t \end{vmatrix} \qquad (2\ 2\ 1)$$

where $\eta_o = \eta \quad 270$ (2 2 2)

a_u a_v a_s are unit vector along u v and s respectively

and

i_t j_t k_t are unit vectors along x y and z respectively



Direction of the incident wave

Fig (1)

Let ω_r be the angular frequency of the dipole and α be the initial angle of the dipole with respect to the u axis Then the instantaneous position of the unit vector $\hat{d}$ is given by

$$d = a_u \cos\psi + a_v \sin\psi \tag{2 2 3}$$

where

$$\psi = \omega_r t + \alpha \tag{2 2 4}$$

From (2 2 1) and (2 2 3)

$$\begin{aligned} d = & -i_t(\sin\eta \cos\psi + \cos\xi \cos\eta \sin\psi) \\ & + j_t(\cos\eta \cos\psi - \cos\xi \sin\eta \sin\psi) + \hat{k}_t(\sin\xi \sin\psi) \end{aligned} \tag{2 2 5}$$

When an RF signal is transmitted by the radar it produces an electric field at the dipole which induces a current in the dipole The dipole reradiates energy due to the current induced in it thus producing an electric field at the radar receiver

Let $\vec{E}_{inc}$ be the field incident on the dipole which is given by

$$\vec{E}_{inc} = E_{OT}\, e_T \tag{2 2 6}$$

where E_{OT} is the magnitude of the electric field and e_T is the t a sm tt polarization vector

If ϕ_o is the polarization angle with respect to the x axis

$$e_T = \hat{i}_t \cos\phi_o + j_t \sin\phi_o \tag{2 2 7}$$

The current den ity J in luced in the dipole is given by

$$J \quad j\,\omega_c\varepsilon_o(\ r \quad 1)\ E_{OT}(e_T\ \hat{d}) \qquad (2\ 2\ 8)$$

where ω_c is the frequency of transmission ε_o is the permittivity of free space and ε_r is the relative permittivity of the medium

The electric field produced by the dipole at a point which is at a distance r from the dipole is

$$\vec{L}_{ref} = \quad \nu^2 \vec{\Pi} \quad + \text{grad}\ (\text{div}\ \vec{\Pi}) \qquad (2\ 2\ 9)$$

where

$$\nu^2 = (\quad \varepsilon\mu \quad \omega_c^2 + j\sigma\ \mu\ \omega_c) \qquad (2\ 2\ 10)$$

μ is the permeability of the medium and

σ is the conductivity of the medium

The Hertz vector $\vec{\Pi}$ is given by [10]

$$\vec{\Pi} \quad \frac{J}{4\pi\ (\sigma + j\,\omega_c\varepsilon)} \quad \frac{e^{-\nu r}}{r}\ d \qquad (2\ 2\ 11)$$

The operations grad and div are

$$\text{grad}\ A \triangleq \nabla\ A \triangleq \frac{\partial A}{\partial x}\hat{i} + \frac{\partial A}{\partial y} \quad j + \frac{\partial A}{\partial z}\hat{k} \qquad (2\ 2\ 12)$$

$$\text{div}\ \vec{A} \triangleq \nabla\ \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \qquad (2\ 2\ 12)$$

From (2 2 8) and (2 2 11)

$$\vec{\Pi} = \frac{j\,\omega_c\ \varepsilon_o(\varepsilon_r \quad 1)}{4\pi\ (\sigma + j\,\omega_c\varepsilon)} \quad E_{OT}\ \frac{e^{\ \nu r}}{r} \quad (e_T\ \hat{d})\ d \qquad (2\ 2\ 13)$$

Rewriting (2 2 5)

$$d = -i_t(\sin\eta\ \cos\psi + \cos\xi\ \cos\eta\ \sin\psi)$$

$$+ j_t(\cos\eta\ \cos\psi\quad \cos\xi\ \sin\eta\ \sin\psi) + \hat{k}_t(\sin\xi\quad \sin\psi)$$

Let

$$A \triangleq \quad \sin\eta\ \cos\psi\quad \cos\xi\ \cos\eta\ \sin\psi$$

$$B \triangleq \cos\eta\ \cos\psi\quad \cos\xi\ \sin\eta\ \sin\psi \tag{2 2 14}$$

$$C \triangleq \sin\xi\ \sin\ \psi$$

Then the unit vector d can be written as

$$d = i_t A + j_t B + k_t C \tag{2 2 15}$$

From (2 2 7) and (2 2 15)

$$d\ e_T = \quad A\cos\phi_o + B\ \sin\phi_o \tag{2 2 16}$$

Let

$$K = \frac{j\,\omega_c\,\varepsilon_o(\varepsilon_r\quad 1)\,E_{OT}}{4\pi\,(\sigma + j\,\omega_c\,\varepsilon)} \tag{2 2 17}$$

and

$$K_1 = K(\ A\cos\phi_o + B\sin\phi_o) \tag{2 2 18}$$

Substituting (2 2 17) and (2 2 18) in (2 2 13)

$$\vec{\Pi} = K_1\,\frac{e^{\nu r}}{r}(\ i_t A + j_t B + k_t C) \tag{2 2 19}$$

Substituting $r = \sqrt{x^2 + y^2 + z^2}$

$$\frac{\partial \Pi_x}{\partial x} = K_1 A \frac{\partial}{\partial x} \left| \frac{e^{\nu \sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2} \right|$$

$$= K_1 A \left| \frac{e^{\nu \sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)} \quad x\left(\nu \quad \frac{1}{x^2+y^2+z^2}\right) \right|$$

imilarly

$$\frac{\partial \Pi_y}{\partial y} = K_1 B \left| \frac{e^{\nu \sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)} \quad y\left(\nu - \frac{1}{\sqrt{x^2+y^2+z^2}}\right) \right| \qquad (2\ 2\ 20)$$

$$\frac{\partial \Pi_z}{\partial z} = K_1 C \left| \frac{e^{\nu \sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)} \quad z\left(\nu - \frac{1}{x^2+y^2+z^2}\right) \right|$$

Therefore

$$\operatorname{div} \vec{\Pi} = K_1 \frac{e^{\nu \sqrt{x^2+y^2+z^2}}}{(x^2+y^2+z^2)} \left(\nu \quad \frac{1}{x^2+y^2+z^2}\right)(xA + yB + zC) \qquad (2\ 2\ 21)$$

$$\operatorname{grad}(\operatorname{div} \vec{\Pi}) = \frac{\partial}{\partial x}(\operatorname{div} \vec{\Pi})\ \hat{i}_t + \frac{\partial}{\partial y}(\operatorname{div} \vec{\Pi})\ \hat{j}_t + \frac{\partial}{\partial z}(\operatorname{div} \vec{\Pi})\ k_t \qquad (2\ 2\ 22)$$

$$\frac{\partial}{\partial x}(\operatorname{div} \vec{\Pi}) = K_1 A \left\{ \frac{e^{\nu r}}{r^2} \left(\nu + \frac{1}{r}\right) - x^2 \left[\frac{e^{\nu r}}{r^2} \quad \frac{x}{r^3} + x\left(\nu + \frac{1}{r}\right)\left(\frac{e^{\nu r}}{r^3} - \frac{2e^{\nu r}}{r^4}\right)\right]\right\} \qquad (2\ 2\ 2)$$

Similarly $\frac{\partial}{\partial y}(\operatorname{div} \vec{\Pi})$ $\frac{\partial}{\partial z}(\operatorname{div} \vec{\Pi})$ can be determined

From (2 2 23) it can be seen that grad $(\operatorname{div}\vec{\Pi})$ contains only

$\frac{1}{r^2}$ $\frac{1}{r^3}$ $\frac{1}{r^4}$ and $\frac{1}{r^5}$ terms Hence grad (div $\vec{\Pi}$) 0 for large values of

$$\text{Ther fore} \quad \vec{E}_{ref} = -\nu^2 \vec{\Pi} = \nu^2 K \frac{e^{-\nu}}{r} (A \cos\phi_o + B \sin\phi_o)$$

$$(\hat{i}_t A + \hat{j}_t B + k_t C) \qquad (2\ 2\ 24)$$

Th electric field received by the radar due to the scattering by the dipole is given by

$$E^s_{rec} \quad E^s_x \cos\phi_r + E^s_y \sin\phi_r \qquad (2\ 2\ 25)$$

wh re ϕ_r is the receiver polarization with re pect to the y axi

$$E^s_x = \nu^2 IA \frac{e^{\nu r}}{r} (A \cos \phi_o + B \sin\phi_o) \qquad (2\ 2\ 26)$$

and

$$E^s_y = \nu^2 KB \frac{e^{-\nu r}}{r} (A \cos \phi_o + B \sin\phi_o)$$

From the above

$$E^s_{rec} = \frac{(\varepsilon\mu\omega_c^2 + j\sigma\mu\omega) j \omega_c \varepsilon_o(\varepsilon_r 1)E_{OT}}{4\pi(\sigma + j\omega_c \varepsilon)} \frac{e^{-\nu r}}{r}$$

$$(A \cos\phi_o + B \sin\phi_o)(A \cos\phi_r \quad B \sin\phi_r) \qquad (2\ 2\ 27)$$

Assuming the medium to be lossless and where the radar is monostatic

$$\sigma = 0 \quad \text{and} \quad \phi_r = \phi_o \qquad (2\ 2\ 28)$$

$$E^{s}_{rec} = K_2 \, E_{OT} \quad \frac{e^{j\omega\sqrt{\mu\varepsilon}\, r}}{r} \left(A \cos\phi_o + B \sin \phi_o \right)^2 \qquad (2\ 2\ 29)$$

where

$$K_2 = \frac{\mu\,\omega_c\,\varepsilon_o(\varepsilon_r - 1)}{4} \qquad (2\ 2\ 30)$$

Radar cross section for the jth scatterer is defined a

$$\sigma_j \triangleq \underset{r_j \to \infty}{Lt} \; 4\pi \; r_j^2 \left| \frac{E^{s}_{ref\ j}}{E_{inc\ j}} \right|$$

From (2 26) and (2 2 19)

$$| E_{inc j} | = E_{OT}$$

$$| E^{s}_{rec\ j} | = \frac{K_2\ E_{OT}}{r_j} \left(A_j \cos\phi_o + B_j \sin\phi_o \right)^2$$

Hence

$$\sigma_j = 4\pi \; K_2^2 \; \left[\left(A_j \cos\phi_o + B_j \sin\phi_o \right)^2 \right]^2 \qquad (2\ 2\ 31)$$

The voltage reflection coefficient is related to the RCS as given below

$$| \bar{V}_j^2 | = \frac{G^2 \, \lambda_c^2}{(4\pi)^3 \, r_j^4} \; | \bar{\sigma}_j | \qquad (2\ 2\ 32)$$

where V_j - is voltage reflection coefficient

and λ_c is the radar wavelength

ubstituting (2 2 31) in (2 2 32)

$$|V_j| = \frac{K_3}{r_j^2} (A_j \cos \phi_o + B_j \quad in \, \phi_o)^2 \qquad (2\ 2\ 33)$$

where

$$K_3 = \frac{G \lambda_c \mu \omega^2 \varepsilon_o (\varepsilon_r \quad 1)}{(4\pi)^2}$$

Therefore

$$\hat{V}_j = \frac{K_3}{r_j^2} (A_j \cos \phi_o + B_j \sin \phi_o)^2 e^{j\beta}$$

where β is the random phase of the reflection coefficient incurrel in th reflection process

It can be shown that

$$(\quad_j \cos \phi_o + B_j \sin \phi_o)^2$$

$$= \hat{C}(\xi_j \quad \eta_j) + \hat{U}(\xi_j \quad \eta_j) e^{j2\psi_j} + \hat{L}(\xi_j \, \eta_j) e^{\quad j2\psi_j}$$

where

$$\psi_j = \omega_{r_j} t + \alpha_j$$

$$\hat{C} = \quad (S^2 \quad R^2 \cos^2 \xi_j)$$

$$\hat{U} = \quad (S + R \cos \xi_j)^2$$

$$L = \quad (\quad R \cos \xi_j)^2$$

$$R = [\cos \phi_o \cos \eta_j + \sin \phi_o \quad in \, \eta_j]$$

$$S = j [\cos \phi_o \sin \eta_j - \sin \phi_o \cos \eta_j]$$

$$\hat{V}_j = \frac{K_3}{r_j^2} [\hat{C}(\xi_j \quad \eta_j) + U(\xi_j \quad \eta_j) e^{j2\psi_j} + \hat{L}(\xi_j \quad \eta_j) e^{\quad j2\psi_j}] e^{j\beta}$$

2.2.2 SCATTERING FUNCTION DERIVATION FOR THE RANDOM DIPOLE MODEL

Let $S_t(t)$ be the complex envelope of the transmitted signal. Then the complex envelope of the received signal from the jth scatterer can be written as

$$r_j(t) = V_j\left(t - \frac{\tau_j(t)}{2}\right) \tilde{S}_t(t - \tau_j(t)) \exp\left[j \omega_c \tau_j(t) + j\beta\right] \quad (2.2.34)$$

where $V_j(t)$ is the reflection coefficient of the jth scatterer, $\tau_j(t)$ is the round trip delay of the jth scatterer, ω_c is the carrier frequency and β is the random phase incurred in the reflection process.

In the beginning of this section it was assumed that the scatterers have an overall drift velocity and also differential velocities. Let v_j be the total velocity of the jth scatterer in the radial direction. The velocities of the scatterers in all the other direction can be neglected since their doppler contribution to the echo signal is very small. Therefore the range of the scatterer at time t is given by

$$r_j(t) = r_{jo} + v_j t \quad (2.2.35)$$

Round trip delay time $\lambda_j(t)$ implies that a signal received at time t was reflected from the scatterer at time $(t - \lambda_j(t)/2)$. At that instant the range of the scatterer was

$$r_j(t - \frac{\lambda_j(t)}{2}) = r_{jo} - v_j(t - \frac{\lambda_j(t)}{2})$$

$$\tau_j(t) \triangleq \frac{2\, r_j(t - \lambda_j(t)/2)}{c}$$

where c is the velocity of light

From the above two eqns

$$\lambda_j(t) = \frac{2\, r_o/c}{1+ v_j/c} - \frac{(2\, v_j/c)t}{1+ v_j/c}$$

assuming that $\frac{v_j}{c} << 1$

$$\lambda_j(t) \approx \frac{2\, r_{jo}}{c} - \frac{2\, v_j}{c}\, t = \lambda_j - \frac{2\, v_j}{c}\, t \qquad (2\ 2\ 36)$$

Assuming that the cloud of scatterers is very far from the radar

$$r_k^2 \approx r_i^2 \approx r_o^2$$

Therefore

$$S_{r_j}(t) = \frac{K_3}{r_o^2} \left[\tilde{\Gamma}(\xi_j, \eta_j) + \tilde{U}(\xi_j, \eta_j)\, e^{j2\left[\omega_{r_j}(t - \frac{\lambda_j(t)}{2}) + \alpha_j\right]}\right.$$

$$\left. + L(\xi_j, \eta_j)\, e^{j2\left[\omega_{r_j}(t - \frac{\lambda_j(t)}{2}) + \alpha_j\right]}\right]$$

$$S_t(t - \lambda_j)\ \exp[\, j\, \omega_c \tau_j(t) + j\, \beta\,] \qquad (2\ 2\ 37)$$

Assuming β to be uniformly distributed in $(0, 2\pi)$ it can be seen that $\tilde{S}_{r_j}$ has zero mean

The total received signal at time t can be written as

$$S_r(t) = \sum_{j=1}^{N_t} \tilde{S}_{r_j}(t) \qquad (2\ 2\ 38)$$

A unin that N_t is a non homogeneous Poisson process with rate $\nu(t)$ the pro ess i stationary and λ_js are the unordered delays the autocorrelation of the re eived ignal which follo from (2 1 11) can be written as

$$R_r(t_\alpha \quad t_\beta \quad \lambda) = R(\,|\,) \quad \nu(\lambda)\ \tilde{S}_t(t_\alpha \quad \lambda)\ S_t^*(t_\beta \quad \lambda) \qquad (2\ 2\ 39)$$

where

$$R_Z(t \quad \tau \quad t \quad \lambda) \quad E\{\frac{K_3^2}{r_o^4}[C(\xi \quad \eta) + U^*(\xi \quad \eta)\ e^{j(2\,\omega_r(t\ \tau \quad \frac{(t\ \tau)}{2}) + 2\alpha)}$$

$$+ L(\xi \quad \eta)\ e^{+j(2\,\omega_r(t - \tau \quad \frac{\lambda(t\ \tau)}{2} + 2\,\alpha)}]$$

$$[C(\xi \quad \eta) + U(\xi \quad \eta) e^{+j(2\,\omega_r(t \quad \frac{\lambda(t)}{2} + 2\,\alpha)}$$

$$+ I(\xi \quad \eta)\ e^{-j(2\,\omega_r(t \quad \lambda\frac{(t)}{2}) + 2\,\alpha)}]\}$$

$$p[\,j\,\omega_c\ \lambda_j(t\ \tau) \quad j\,\omega_c\ \lambda_j(t)]$$

and Z_js are assumed to be uncorrelated and identically d s tributed

$$R_{\tilde{Z}}(t \quad \tau \quad t \quad \lambda) = E\{\frac{\zeta_3^2}{r_o^4}\ [\,|C^2(\xi \quad \eta)| + |J^2(\xi \quad \eta)|\ e^{+j2\omega_r\tau}$$

$$+ |L^2(\xi \quad \eta)|\ e^{j2\,\omega_r\tau}]\ \exp(j\,\omega_d\tau)\}$$

(2 2 40)

where $\omega_d = \dfrac{2\,\omega_c\ v_j}{c}$

and $\dfrac{\omega_r\ v_j}{c} \ll 1$

Therefore

$$R_Z(\tau) = \frac{K_3^2}{r_o^4} \iiiint_{\infty}^{\infty} p(\omega_d)\, p(\omega_r)\, p(\xi)\, p(\eta)\, d\omega_d\, d\omega_r\, d\xi\, d\eta$$

$$[\,|\ ^2(\xi\ \eta)| + |U^2(\xi\ \eta)|\ e^{+j2\omega_r\tau} +$$

$$+ |L^2(\xi\ \eta)|\ e^{\ j2\omega_r\tau}]\ \exp(j\omega_d\tau) \qquad (2\ 2\ 41)$$

where ω_d ω_r ξ anl η are a sumed to be independent

The scatterers can move towards or aw y from th radar along the lin of sight with equal probability and with their velocity di tributions centred about $\pm\overline{\omega}_d$ Let $P(E)$ be the probability of the event ω_d being positive and $P(\overline{E})$ is the probability of the event ω_d being negative

Since these probabilities are equal $P(E) - P(\overline{E}) = \frac{1}{2}$

Let $\omega_d - X + \overline{\omega}_d$

$$P\omega_d\ [\omega_d < \omega_d] = P\omega_{d/E}\ [\omega_d < \omega_d/\ (E)]\ P(E)$$

$$+ P\omega_{d/\overline{E}}\ [\omega_d < \omega_d/\ (\overline{E})]\ P(\overline{E})$$

$$F\omega_d(\omega_d) - \quad F_X[\omega_d \quad \overline{\omega}_d] + \tfrac{1}{2} F_X[\omega_d + \overline{\omega}_d]$$

$$p\omega_d(\omega_d) = \tfrac{1}{2}\ p_X(\omega_d \quad \overline{\omega}_d) + \quad p_X(\omega_d + \omega_d)$$

$$\int_{\infty}^{\infty} p_{/d}(\omega_d)\ e^{+j\omega_d} \quad - \tfrac{1}{2}\int_{\infty}^{\infty} p_{/}(\omega_d \quad \omega_d)\ e^{+j\omega_d \tau}\ d\omega_d$$

$$+ \int_{\infty}^{\infty} p_{/}(\omega_d + \bar{\omega}_d)\ e^{j\omega_d \tau}\ d\omega_d$$

$$= \cos(\bar{\omega}_d\ \tau)\ \phi\omega_d(\tau) \qquad (2\ 2\ 42)$$

Assumin_ t 1at each orientation of the rotation axis is equally probable

$$< X^2 > = \int_0^{2\pi}\int_0^{\pi} Y^2 \sin\xi\ d\xi\ d\eta \qquad (2\ 2\ 43)$$

where Y^2 repre ent $|\ ^2(\xi\ \eta)|$ $|\ U^2(\xi\ \eta)|$ and $|\ L^2(\xi\ \eta)\ |$

From the above (2 2 41) reduces to

$$R_{x}(\tau) = \frac{K_3^2}{r_0^4}[< \sigma^2 > + < U^2 > \phi\omega_r(2\tau) + < L^2 > \phi\omega_r(\ 2\tau\ )]\phi\omega_d(\tau)\ \cos(\omega_d \tau)$$

(2 2 45)

where

$$\phi\omega_r(\tau) - \int_{\infty}^{\infty} p(\omega_r)\ e^{j\omega_r \tau}\ d\omega_r \qquad (2\ 2\ 46)$$

From (2 1 13) and (2 2 45)

$$R_x(\tau\ \lambda) = \frac{K_3}{r_o^4}[< \sigma^2 > + < U^2 > \phi\omega_r(2\tau)$$

$$+ < L^2 > \phi\omega_r(\ 2\tau)]\ \phi\omega_d(\tau)\ \cos(\omega_d\ \tau)\ \nu(\lambda) \qquad (2\ 2\ 47)$$

Hence the normalised correlation function is

$$g(\tau\ \lambda) = \frac{1}{<b^2>}\ [\ <C^2> + <U^2>\ \phi\omega_r(2\tau) + <L^2>\phi\omega_r(2\tau)\,]$$

$$\phi\omega_d(\tau)\ \cos(\bar{\omega}_d\tau) \qquad (2\ 2\ 48)$$

where $<b^2> \quad = <C^2 + U^2 + L^2>$ (2 2 49)

From (2 2 48) the scattering function can be written as

$$S(f\ \lambda) = \int_{\infty}^{\infty} g(\tau\ \lambda)\ e^{-j2\pi f\tau}\, d\tau \qquad (2\ 2\ 50)$$

2 2 3 SCATTERING FUNCTION CALCULATION FOR GAUSSIAN CASE

In this sub section scattering function is calculated assuming that the doppler frequency ω_d and the angular frequency ω_r have Gaussian probability distributions

Let

$$p_{\omega_d}(\omega_d) = \frac{1}{\sqrt{2\pi}\sigma_d}\ \exp\ [\ (\omega_d - \bar{\omega}_d)^2\ /2\sigma_d^2\ ] \qquad (2\ 2\ 51)$$

and

$$p_{\omega_r}(\omega_r) = \frac{1}{\sqrt{2\pi}\sigma}\ \exp\ [\ (\omega_r \quad \bar{\omega}_r)\ /2\sigma_r^2\ ] \qquad (2\ 2\ 52)$$

$$\psi_{\omega_d}(\tau) \quad \exp\ (j\bar{\omega}_d\tau)\ \exp(\ \tfrac{1}{2}\sigma_d^2\tau^2) \qquad (2\ 2\ 53)$$

$$\psi_{\omega_r}(\tau) = \exp\ (j\,\bar{\omega}_r\tau)\ \exp(\ \tfrac{1}{2}\ \sigma_r^2\ \tau^2) \qquad (2\ 2\ 54)$$

The values of $<C^2>$ $<U^2>$ and $<L^2>$ are calculated in [1] for the following special cases 1) linear transmit linear receive 2) circular transmit circular receive

3) orthogonal linear tran mit and receive 4) orthogonal circular tran m t and rec ive They are given as follows

Case	$< C^2 >$	$< U^2 >$	$< L^2 >$	$< b^2 >$
1	$\frac{2}{15}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{5}$
2	$\frac{1}{30}$	$\frac{1}{20}$	$\frac{1}{20}$	$\frac{2}{15}$
3	$\frac{1}{60}$	$\frac{1}{40}$	$\frac{1}{40}$	$\frac{1}{15}$
4	$\frac{7}{60}$	$\frac{1}{120}$	$\frac{1}{120}$	$\frac{2}{15}$

From the above it can be seen that $< U^2 > = < L^2 >$

Substituting (2 2 53) and (2 2 54) in (2 2 48)

$$g(\tau\ \lambda) = \frac{1}{\langle b^2 \rangle} [< C^2 > + 2 < S^2 > \exp(2\ \sigma_r^2\ \tau^2)\ \cos(2\ \bar{\omega}_r \tau)]$$

$$\cos(\bar{\omega}_d \tau)\ \exp\ (\ \tfrac{1}{2}\ \sigma_d^2\ \tau^2) \qquad (2\ 2\ 55)$$

where $< L^2 > = < L^2 > - < \ ^2 >$

The phase factor $e^{j\omega_d \tau}$ is suppressed because it along with $e^{j\ \omega_c \tau}$ corre pond to the hetrodyming of the received signal with an o cillator of frequency $(\omega_c + \bar{\omega}_d)$

$$S(f\ \lambda) - \int_{\infty}^{\infty} \frac{1}{< b^2 >} [< C^2 > + 2 < S^2 > \exp(-2\ \sigma_r^2\ \tau^2)\ \ \cos(2\ \bar{\omega}_r \tau)]$$

$$\cos(\omega_d \tau)\ \exp\ (\ \tfrac{1}{2}\ \sigma_d^2\ \tau^2)\ \ e^{-j2\pi f \tau}\ \ d\tau$$

(2 2 56)

SECTION 2 3 LXA TILS

In this ection some of the result simulated u ing the model discu sed in previous section are compared with tho e obtained experimentally In the pa t e pcriments were conducted to obtain the frequency spectrum of po cr flu tuations where the power ret irn P(t) was treated a a r ndom process b cause of the random fluctuations of the scatterers The correlation function of the power return ie found to be [9]

$$\rho(\tau) = \frac{\overline{P(t)\ P(t+\tau)} \quad (\overline{P})^2}{\overline{P^2(t)} \quad (\overline{P})^2} = g^2(\tau) \qquad (2\ 3\ 1)$$

where $g(\tau)$ is the coi elation function of the echo voltage

$\rho(\tau)$ was d termined by the interpolation of the di crete obs rvation and the power pectrum which is the fourier transforn of $\rho(\tau)$ was plott d For comparison it is nece ssary to find out $\rho(\tau)$ from the results obtained in Section 2 1r m (2 2 55) $g(\tau)$ is given by

$$g(\tau) = \frac{1}{<b^2>}[<C^2> + 2<S^2> \exp(-2\sigma_r^2\tau^2)\cos(2\bar{\omega}_r \quad)]$$

$$\cos(\bar{\omega}_d\tau)\exp(\ \tfrac{1}{2}\sigma_d^2\tau^2) \qquad (2\ 3\ 2)$$

$$\rho(\tau) = [\frac{<C^2>}{<b^2>} + \frac{2<S^2>}{<b^2>} \exp(\ 2\sigma_r^2\tau^2)\cos(2\bar{\omega}_r\tau)]^2$$

$$\cos^2(\bar{\omega}_d\tau\ )\exp\ (-\sigma_d^2\tau^2) \qquad (2\ 3\ 3)$$

$$S_1(f) = \int_{\infty}^{\infty} \rho(\tau)\, e^{j2\ \ f\tau}\, d\tau \qquad (2\ 3\ 4)$$

$S_1(f)$ in eqn (2 3 4) is simulated and is compared with a few power spectra obtained experimentally

In 1949 E J Barlow [6] reported spectra for wooded hill ea echo rain clouds and haff m asured at a fr quency 1 GHz According to Barlow lutter po/er spectra c 1 be r presented by Gau sian shaped curves of the form

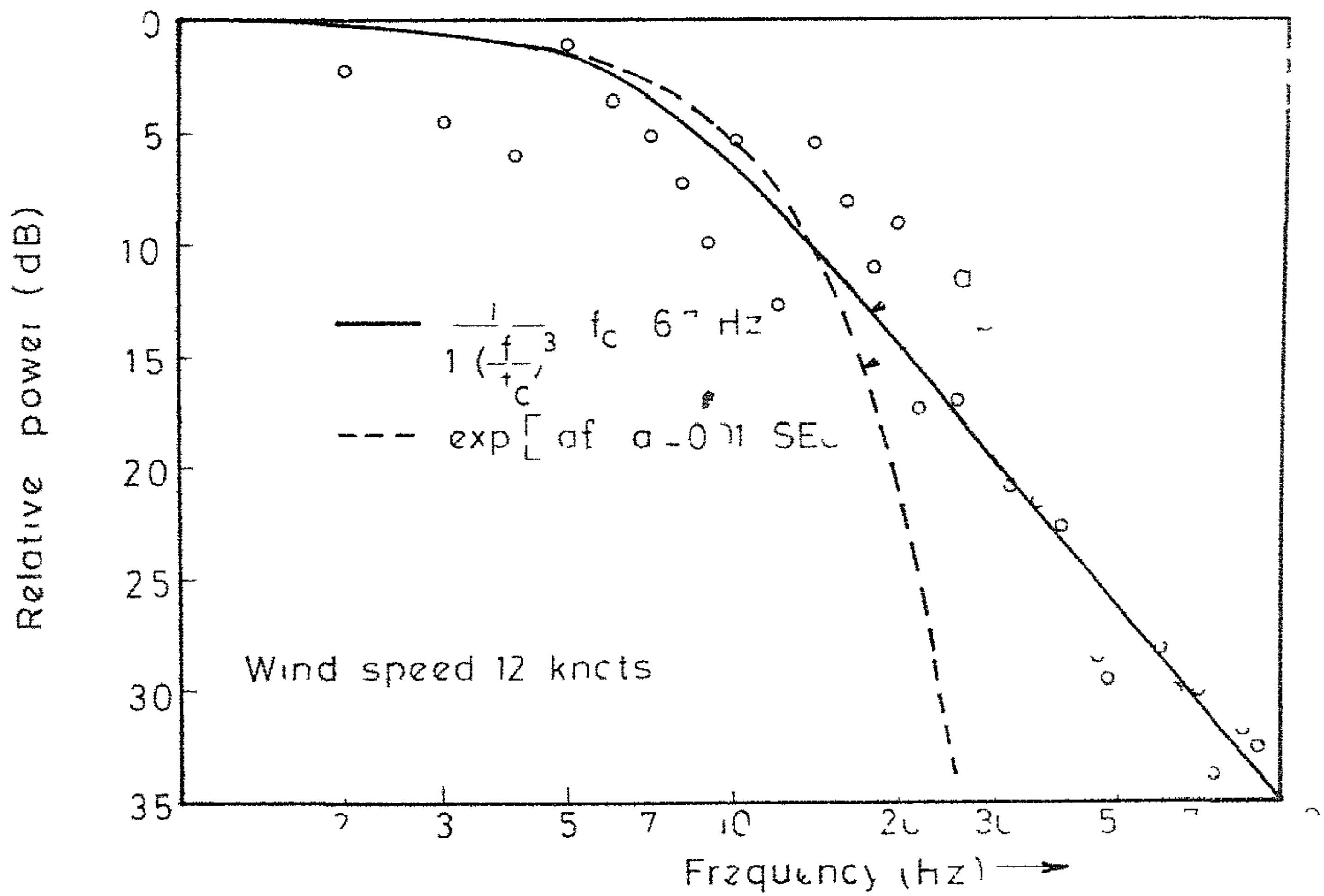
$$\,^{\tau}(f) = \exp\ [\ \ a\ (f/f_o)^2]\qquad (2\ 3\ 5)$$

where f_o is th radar transmitter frequency and a i a p rameter dependent on the radar target type In practice it was ol erv d by i hbein Grav line et al that the po/r spectrum does not t ke Gau sian shape According to th n a simple expre sion that gives good agreement with their measured spectra for deciduous foliage is

$$P(f) = \frac{1}{1 + (f/f_c)^3}\qquad (2\ 3\ 6)$$

where $f_c - 1\ 33\ \ e^{0\ 13,6\ v}$ and v is th ind peed in knots

For their exp riment on tree echo in 12-knot wind Fi hbein Graveline et al used an x band coherent pulse radar with horizontal polarization and the experimental results are shown by curve a in Fig (2) The Gaussian shaped curve b illustrates the results calculated using eqn (2 3 5) From



Source F shbein Gravel ne and R ttenborn 19h7

Fig 2 Power spectrum obtained with an X band radar Eo t so c c rep usu t r measurement

Fig (2) it can be clearly s en that the Gaussian d curve falls off fa ter at higher frequencies than d es the experimental curve ll the result are simulated fo linear polarization and the simulation i done by trial and error

EXAMPLE 1

Here an attempt is made to simulate results predict d by L J Barlow In Fig (3) curve a b are the results reported by E T Barlo and th curves a b illustrate the si ulated results Th parameters for th curves a and b are as follows

For th curve - a

Mean value of th doppler shift - $\overline{\omega}_d$ = 5 3 rad/sec

Variance of the doppler shift = σ_d = 0 15 rad/ ec

Mean value of the angular frequency $\overline{\omega}_r$ = 2 4 rad/s c

Variance of the angular frequency σ_r = 1 2 rad/sec

For the curve b

$\overline{\omega}_d$ 13 3 rad/sec

σ_d - 10 9 rad/sec

$\overline{\omega}_r$ = 11 0 rad/s c

σ_r = 9 0 rad/sec

From Fig (3) it can be seen that the simulated result do not fall off fast at higher frequency This is in upport of the conclusions made by Fishbien Therefore it can be inferred that th simulated re ults are in agreement with the experimental results

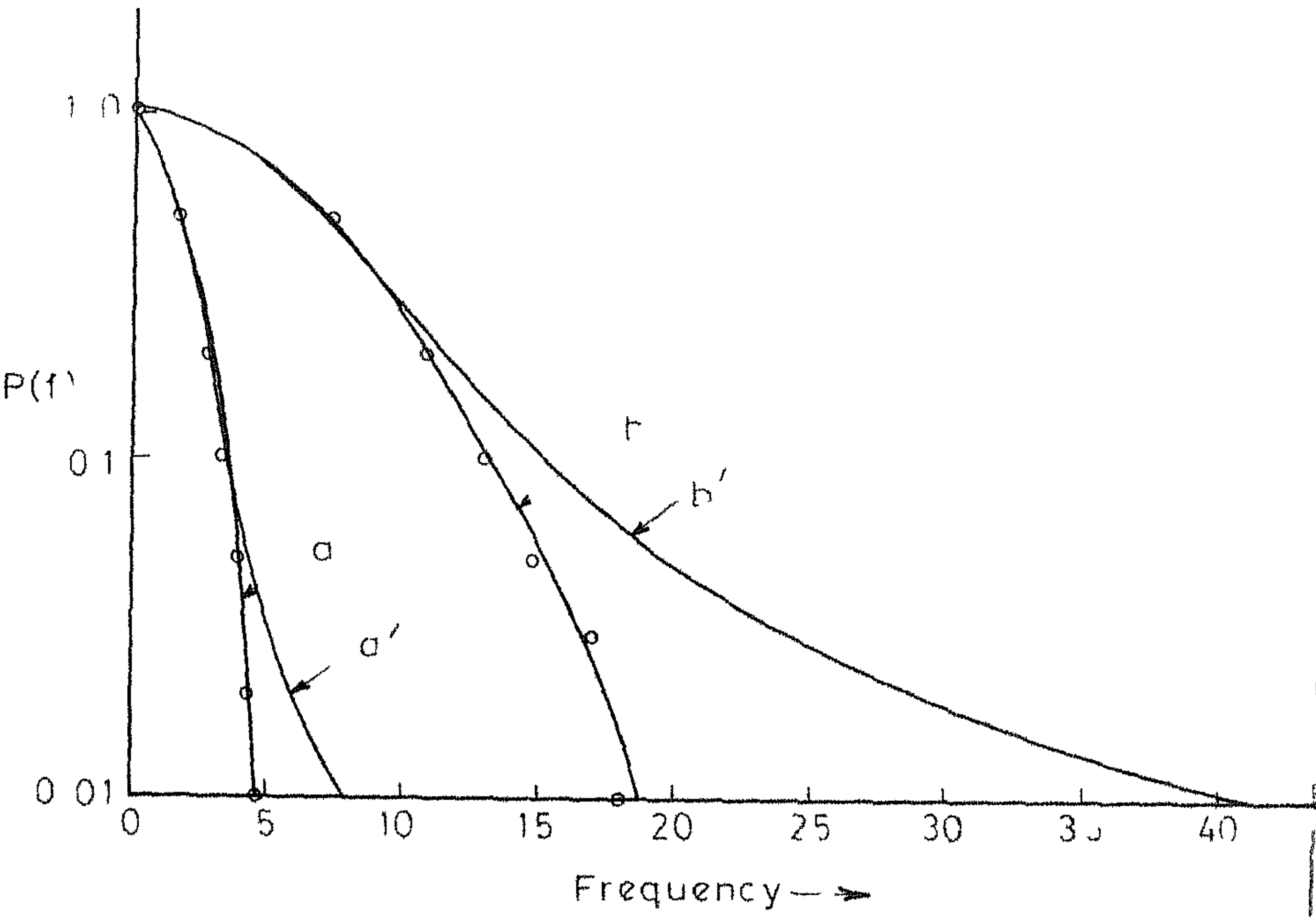


Fig 3 Frequency spectra of various types of complex targe
at 1 GHz

EXAMPLE 2

In Fig (4) curve a illustrates the experimental results reported in [9] It show the spectra for the echo of chaf ut for $\lambda_c = 10$ cm as measured on $\lambda_e - 9\ 2$ cm The data were taken 3 min after the chaff was dispen ed from a low movin blimp The r sult of simulation re ho m by the curve b and the value of parameter n simulation are

$$\bar{\omega}_d = 15\ 5 \text{ rad/sec}$$

$$\sigma_d - 16\ 0 \text{ rad/sec}$$

$$\bar{\omega}_r = 15\ 0 \text{ rad/sec}$$

$$\sigma_r - 2\ 0 \text{ rad/sec}$$

EXAMPLE 3

Simulated result of the model suggested by J L Wong I S Reed t al are shown by curve - b in Fig (5) Curve - a repres nts th experimental results and curve c illustrates the simulat d results using the pre ent mod l In [1] the positive and negative velocities of the scatterers were not con idered From the Fig (5) it can seen that the curve c i closer to the curve a than the curve - b

In all the above three examples it was seen that the results simulated using the model sugge ted in this work are closer to the experimental result obtained than the

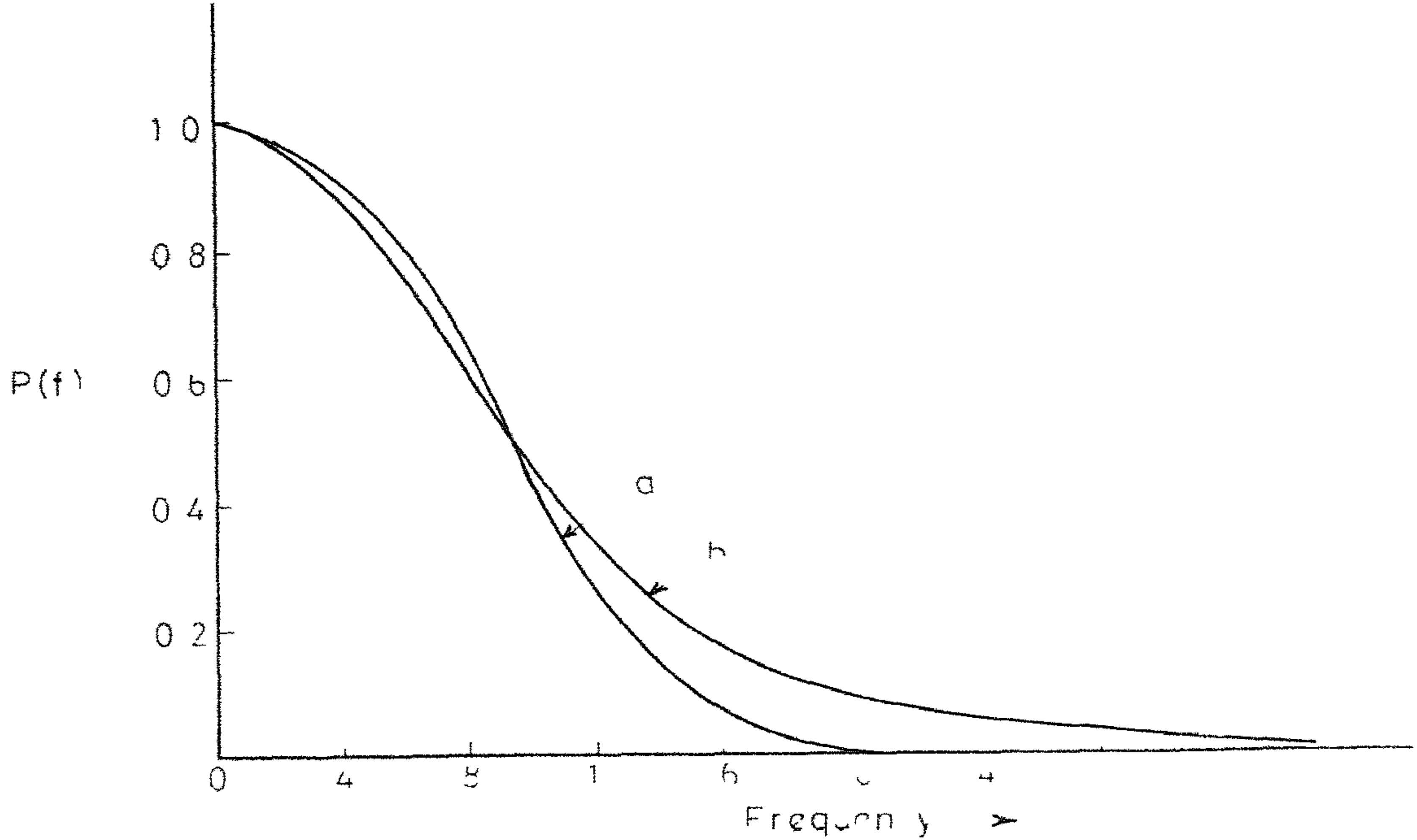


Fig 4 Frequency spectrum for the cn ff cut or λ_c 0 n r

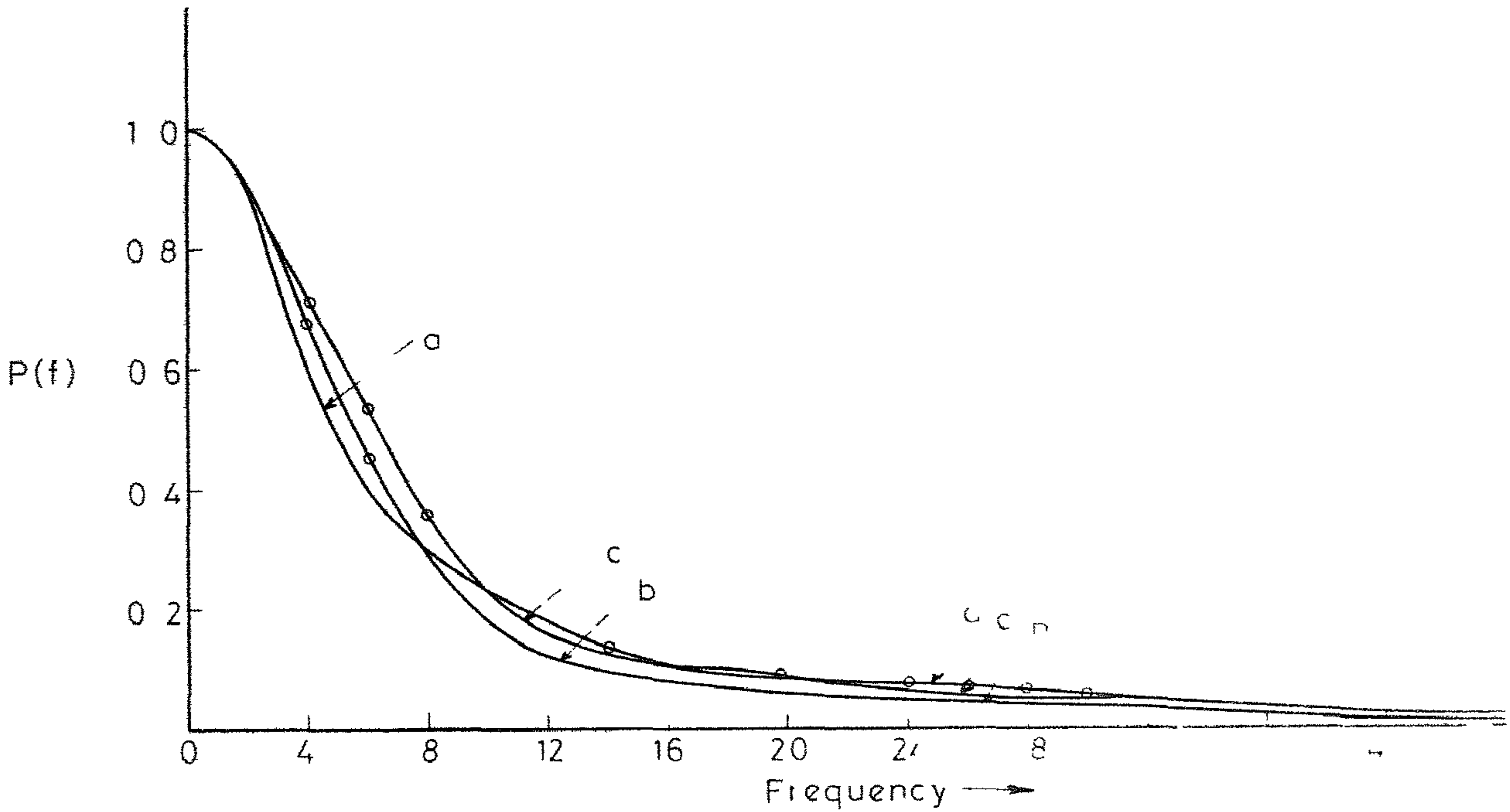


Fig 5 Function frequency spectrum for ground clut[illegible] terrain at wind speed 50 mph

previous models The simulation was don by trial and error method Better results c n be obtained by using gradi nt method or minimum mean square estimate

CHAPTER III

FLUCTUATING ELLIPSOIDAL SCATTERER MODEL

In Chapter 2 an expre sion for the eatterin function was derived for an ensemble of random number of point scatterers whose returns follow a non homogeneous Poisson distribution In this chapter an attempt is made to derive the scattering function for an extended clutter target which following the model sugge ted by the work of J W W ight [2] is treated as a finite collection of suitably located ellipsoidal scatterers with varying cross sections A detailed description of the model along with the derivation of the scattering function is given in Section 3 1 Section 3 2 contains the calculation of the scattering fun ction for a speci l case

3 1 1 DESCRIPTION OF THE MODEL

As in [2] the illuminated volume of the scatterers i divided into a finite number of ellipsoids Each ellipsoid is associated with a suitable modulating function to take care of the irregularities in shapes and also to account for the shadowing effects A point on the surface of an ellipsoid which has the direction cosines same as those of the line of sight and which is in the same quadrant as the line of sight is hosen as the representative point of the ellipsoid The radar cross section of the representative point (henceforth referred to as specular point) is equal to the RCS of the ellipsoid calculated at the specular point The advantage of choosing

ellipsoids and representing each of them by a single point is that a large number of scatterers in the ellipsoid are represented by a single point thus making the effective number of scatterers in the illuminated volume a finite number

Three coordinate systems are choosen to describe the ellipsoids First is the radar fixed coordinate system (r c s) second is the target fixed coordinate system (t c s) which is parallel to the first and the third is the local coordinate system (l c s) The transformation between the first two coordinate systems is linear and therefore the aspect angles of one with respect to the other are equal The l c s is the ellipsoid own coordinate system with its centre coinciding with that of the ellipsoid The axes of the l c s are the axes of the ellipsoid

The vectorial representation of the three coordinate systems in Fig (6) is as follow

$$\overrightarrow{O_r P} = \hat{i}_r x + \hat{j}_r y + \hat{k}_r z$$

$$\overrightarrow{O_t P} = \hat{i}_t x + \hat{j}_t y + \hat{k}_t z$$

$$\overrightarrow{O_l P} = \hat{i}_l x_l + \hat{j}_l y_l + \hat{k}_l z_l \qquad (3\ 1\ 1)$$

$$\overrightarrow{O_r O_t} = \hat{i}_r x_o + \hat{j}_r y_o + \hat{k}_r z_o$$

$$\overrightarrow{O_t O_l} = \hat{i}_t x_{tl} + \hat{j}_t y_{tl} + \hat{k}_t z_{tl}$$

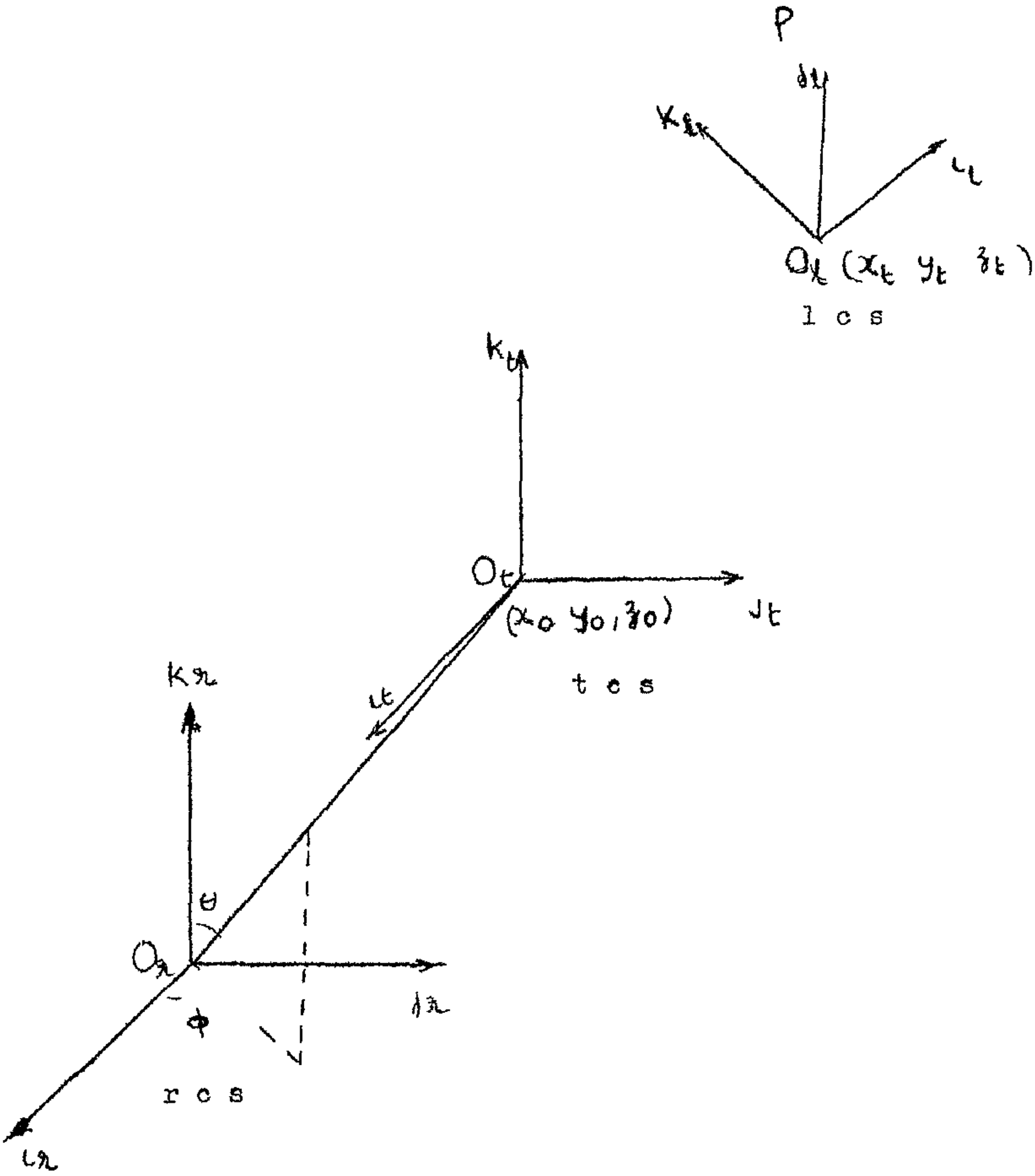


Fig 6

The relationship between the t c s and r c s is given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x & x_o \\ y & y_o \\ z & z_o \end{bmatrix} \qquad (3\ 1\ 2)$$

To allow for arbitrary orientations of the ellipsoids we consider translational and rotational transformations of the t c s to obtain the l c s After an initial translational transformation to the centre of the ellipsoid two rotational transformations are followed by such that the axes of the l c s coincide with the axes of the ellipsoid In the rotational transformation the axes are rotated initially by an angle ϕ_t about the z axis and then by an angle $_t$ about the new y axis The coordinates of a point in the target fixed coordinate system and the local coordinate system are related by

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = [\,T_i\,] \begin{bmatrix} x & x_{ti} \\ y & y_{ti} \\ z & z_{ti} \end{bmatrix} \qquad (3\ 1\ 3)$$

wh re

$$[\,T_i\,] = \begin{bmatrix} \cos\phi_{ti}\ \cos\theta_{ti} & \sin\phi_{ti}\ \cos\theta_{ti} & \sin\theta_{ti} \\ \sin\phi_{ti} & \cos\phi_{ti} & 0 \\ \cos\phi_{ti}\ \sin\theta_{ti} & \sin\phi_{ti}\ \sin\theta_{ti} & \cos\theta_{ti} \end{bmatrix}$$

(3 1 4)

where ϕ_{ti} and θ_{ti} are right handed angles and the sub script i indicates the ith l c s

LIST OF SYMBOLS

r c s		radar fixed coordinate system
t c s	-	target fixed coordinate system
l c s		local coordinate system
$(x_o\ y_o\ z_o)$	-	origin of the t c s in the r c s
$(x_{ti}\ y_{ti}\ z_{ti})$		origin of the l c s in the t c s
$(x\ \ y\ \ z\ )$	-	coordinates of a point in the r c s
$(x\ y\ z)$		coordinates of a point in the t c s
$(x_i\ y_i\ z_i)$	-	coordinates of the specular point of the ith ellipsoid in the l c s
$\theta\ \ \phi$	-	aspect angles of the r c s in the t c s
$\theta_{li}\ \ \phi_{li}$	-	aspect angles of the centre of the ith l c s in the t c s
$\theta_i\ \ \phi_i$		aspect angles of the ith specular point in the ith l c s
$\theta_{ti}\ \ \phi_{ti}$	-	describe the transformation between l c s and t c s
R_o		distance between the centre of the r c s and the centre of the t c s
$a_i\ b_i\ c_i$	-	semi axes of the ith ellipsoid
M_i		modulating function of the ith ellipsoid
$R_{1i}\ R_{2i}$		Radii of curvature of the ith ellipsoid at its specular point
V_i		voltage reflection coefficient of the ith scatterer
σ_i		RCS of the ith scatterer
v_i		linear velocity of the ith scatterer

τ_i delay of the ith scatterer

λ_c transmitted wave length

c velocity of light

NOTE ith specular is referred to as the ith scatterer

3 1 2 DETERMINATION OF THE VOLTAGE REFLECTION COEFFICIENT

The RCS of the ith ellipsoid is given by [2]

$$S_i \quad M_i \pi R_{1i} R_{2i}$$

where M_i is the modulating function of the ith ellipsoid R_{1i} and R_{2i} are the radii of curvature of the ith ellipsoid at its specular point We calculate the coordinates of the specular point and the product of the radii of curvature as below

Let

$$F_i(x\ y\ z) = \frac{x^2}{a_i^2} + \frac{y^2}{b_i^2} + \frac{z^2}{c_i^2} - 1 = 0 \qquad (3\ 1\ 6)$$

be the equation of the ith ellipsoid with semi axes a_i b_i and c_i The normal to the ellipsoid at $(x_i\ y_i\ z_i)$ is given by

$$\vec{N} - \frac{2x_i}{a_i^2}\hat{i}_i + \frac{2y_i}{b_i^2} j_i + \frac{2z_i}{c_i^2} k_i \qquad (3\ 1\ 7)$$

From the above the direction cosines of the normal are

$$\cos\delta_x = \frac{x_1}{a_1^2 r}$$

$$\cos\delta_y = \frac{y_1}{b_1^2 r} \qquad (3\ 1\ 8)$$

$$\cos\delta_z = \frac{z_1}{c_1^2 r}$$

where

$$r = \sqrt{\frac{x_1^2}{a_1^4} + \frac{y_1^2}{b_1^4} + \frac{z_1^2}{c_1^4}}$$

The direction cosines of the line or sight in the target coordinate system are

$$\cos\delta_x = \sin\theta\ \cos\phi$$

$$\cos\delta_y = \sin\theta\ \sin\phi \qquad (3\ 1\ 9)$$

$$\cos\delta_z = \cos\theta$$

But th direction cosines of the line of sight in 1 c are

$$\cos\delta_x = \sin\theta_1\ \cos\phi_1$$

$$\text{os}\ \delta_y = \sin\theta_1\ \sin\phi_1 \qquad (3\ 1\ 10)$$

$$\text{co}\ \delta_z = \cos\theta_1$$

where

$$\begin{bmatrix} \sin\theta_1 \cos\phi_1 \\ \sin\theta_1 \sin\phi_1 \\ \cos\theta_1 \end{bmatrix} = [T_1] \begin{bmatrix} \sin\theta \cos\phi \\ \sin\theta \sin\phi \\ \cos\theta \end{bmatrix} \qquad (3\ 1\ 11)$$

The normal to the ellipsoid at the specular point should have the same direction cosines as the line of sight and should be in the same quadrant as the line of sight From the above equations the parametric angular coordinate u_s and v_s of the specular point are obtained as

$$v_{si} = \tan^{-1}\left\{\frac{b_1}{a_1} \tan\phi_1\right\} \qquad (3\ 1\ 12)$$

$$u_{si} = \tan^{-1}\left\{\frac{1}{c_1} (\tan\theta_1)\sqrt{a_1^2 \cos^2\phi_1 + b_1^2 \sin^2\phi_1}\right\}$$

$R_{1i}\ R_{2i}$ as given in [2] is

$$R_{1i}\ R_{2i} = \left|\frac{\left(\frac{\partial F_1}{\partial x_1}\right)^2 + \left(\frac{\partial F_1}{\partial y_1}\right)^2 + \left(\frac{\partial F_1}{\partial z_1}\right)^2}{\Delta}\right| \qquad (3\ 1\ 13)$$

where ith specular point

$$= \begin{vmatrix} \frac{\partial^2 F_1}{\partial x^2} & \frac{\partial^2 F_1}{\partial x \partial y} & \frac{\partial^2 F_1}{\partial x \partial z} & \frac{\partial F_1}{\partial x} \\ \frac{\partial^2 F_1}{\partial x \partial y} & \frac{\partial^2 F_1}{\partial y^2} & \frac{\partial^2 F_1}{\partial y \partial z} & \frac{\partial F_1}{\partial y} \\ \frac{\partial^2 F_1}{\partial x \partial z} & \frac{\partial^2 F_1}{\partial y \partial z} & \frac{\partial^2 F_1}{\partial z^2} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} & 0 \end{vmatrix}$$

Hence $R_{1i} R_{2i} - a_i^2 b_i^2 c_i^2 (\frac{x_i}{a_i^4} + \frac{y_i}{b_i^4} + \frac{z_i}{c_i^4})^2$ (3 1 14)

where (x_i y_i z_i) are the coordinates of the sp cular point in the l c and they are determined by equating (3 1 8) and (3 1 10)

$$x_i = a_i \sin u_{si} \cos v_{si} = \frac{a_i^2}{P_i} \sin \theta_i \cos \phi_i$$

$$y_i = b_i \sin u_{si} \sin v_{si} = \frac{b_i^2}{P_i} \sin \theta_i \sin \phi_i \qquad (3\ 1\ 15)$$

$$z_i = c_i \cos u_{si} = \frac{c_i^2}{P_i} \cos \theta_i$$

where $P_i^2 = a_i^2 \sin^2 \theta_i \cos^2 \phi_i + b_i^2 \sin^2 \theta_i \sin^2 \phi_i + c_i^2 \cos^2 \theta_i$

(3 1 1)

From the above it can be seen that θ_i and ϕ_i are also the aspect angl s of the ith specular point

S_i the RCS of th ith ellipsoid is determined from (3 1) (3 1 14) and (3 1 15)

$$S_i = M_i \pi R_{1i} R_{2i} = \frac{M_i \pi a_i^2 b_i^2 c_i^2}{P_i^4} \qquad (3\ 1\ 17)$$

To derive th scattering function it is also necessary to d termin th phase of the scatt red electromagn tic field at the r ceiver Th relative phase difference b tween the reflected electromagnetic waves of two scatterers is given by

$$\alpha_i = \frac{4\pi}{\lambda_c} \Delta R_i \qquad (3\ 1\ 18)$$

where ΔR_i is the difference in range

The phase of the electric field scattered by the ith scatterer can be determined by using the target coordinate centre as the reference point. Using this

$$\Delta R_i = R_o \quad R_i - x^i \sin\theta \cos\phi \quad y^i \sin\theta \sin\phi \quad z^i \cos\theta$$

(3 1 19)

where R_o is the distance between the radar and the target coordinate systems, R_i is the distance between the ith specular point and the r c s and x^i y^i z^i denote the coordinates of the specular point in t c s

From (3 1 18) and (3 1 19)

$$\alpha_i = \frac{4\pi}{\lambda_c} [-x^i \sin\theta \cos\phi \quad y^i \sin\theta \quad \sin\phi \quad z^i \cos\theta]$$

$$= \frac{4\pi}{\lambda_c} \begin{bmatrix} x^i \\ y^i \\ z^i \end{bmatrix}^t \begin{bmatrix} \sin\theta & \cos\phi \\ \sin\theta & \sin\phi \\ \cos\theta & \end{bmatrix}$$

From (3 1 3)

$$\begin{bmatrix} x^i \\ y^i \\ z^i \end{bmatrix} = [T_i]^{1} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \begin{bmatrix} x_{ti} \\ y_{ti} \\ z_{ti} \end{bmatrix}$$

therefore

$$\alpha_i = \frac{4\pi}{\lambda_c} \left\{ \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^t [T_i^{1}]^t + \begin{bmatrix} x_{ti} \\ y_{ti} \\ z_{ti} \end{bmatrix}^t \right\} \begin{bmatrix} \sin\theta & \cos\phi \\ \sin\theta & \sin\phi \\ \cos\theta & \end{bmatrix}$$

Since $[T]$ is an orthogonal matrix

$$[T_i^{-1}]^t = [T_i] \qquad \forall \; i$$

Hence

$$\alpha_i = \frac{4\pi}{\lambda_c}\left\{\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^t [T_i] \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} + \begin{bmatrix} x_{ti} \\ y_{ti} \\ z_{ti} \end{bmatrix}^t \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}\right\}$$

From (3 1 11)

$$\alpha_i = \frac{4\pi}{\lambda_c}\left\{\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^t \begin{bmatrix} \sin\theta_i\cos\phi_i \\ \sin\theta_i\sin\phi_i \\ \cos\theta_i \end{bmatrix} + \begin{bmatrix} x_{ti} \\ y_{ti} \\ z_{ti} \end{bmatrix}^t \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix}\right\}$$

$$= \frac{4\pi}{\lambda_c}\left[(x_i\sin\theta_i\cos\phi_i + y_i\sin\theta_i\sin\phi_i + z_i\cos\theta_i) + (x_{ti}\sin\theta\cos\phi + y_{ti}\sin\theta\sin\phi + z_{ti}\cos\theta)\right]$$

Substituting for x_i y_i z_i from (3 1 15)

$$\alpha_i = \frac{-4\pi}{\lambda_c}\left[\frac{1}{P_i}\left(a_i^2\sin^2\theta_i\cos^2\phi_i + b_i^2\sin^2\theta_i\sin^2\phi_i + c_i^2\cos^2\theta_i\right) + (x_{ti}\sin\theta\cos\phi + y_{ti}\sin\theta\sin\phi + z_{ti}\cos\theta)\right]$$

From (3 1 16)

$$\alpha_i = \frac{4\pi}{\lambda_c}\left[P_i + (x_{ti}\sin\theta\cos\phi + y_{ti}\sin\theta\sin\phi + z_{ti}\cos\theta)\right] \qquad (3\ 1\ 17)$$

Therefore the reflected field from the ith scatterer has a phase dependence $e^{j\alpha_i}$ relative to the target coordinate system. The voltage reflection coefficient of the ith scatterer can be

calculated using the R S of the same The relationship between the above two is given by

$$|\bar{V}_i^2| = \frac{G^2 \lambda_c^2}{(4\pi)^3 R_i^4} |\bar{\sigma}_i| \qquad (3\ 1\ 20)$$

From (3 1 17) σ_i is

$$\sigma_i = \frac{M_i \pi a_i^2 b_i^2 c_i^2}{p_i^4} \qquad (3\ 1\ 21)$$

therefore

$$|V_i| = \frac{G \lambda_c M_i}{(4\pi)^{3/2} R_i^2} \sqrt{\frac{a_i^2 b_i^2 c_i^2}{P_i^4}} \qquad (3\ 1\ 22)$$

By taking into account the phase term and assuming that the scatterers are very far from the radar such that $R_i^2 = R_j^2 - R_o^2$ $\hat{V}_i$ may then be expressed as

$$\hat{V}_i = \frac{G \lambda_c}{8 \pi R_o^2} M_i \frac{a_i b_i c_i}{P_i^2} e^{j(\alpha_i + \beta)} \qquad (3\ 1\ 23)$$

in which we have also associated a random phase shift β with the reflection process

3 1 3 DERIVATION OF THE SCATTERING FUNCTION

The scatterers under the effects of wind forces have linear and rotational movements If the wind velocities are large the movement of the scatterers causes variations in the echo signal which can not be neglected In the present model to account for the random motions of the scatterers it is assumed that the ellipsoids have random fluctuations about

their mean positions and linear velocities v_i along the line of sight. This implies that the local coordinate system has random fluctuations and consequently θ_t and ϕ_t are random processes. The size of the ellipsoid is also assumed to change randomly thus making the semi axes a, b and c random processes. Then

$$\tilde{V}_i(t) = \frac{G \lambda_c}{8 \pi R_o^2} \; M_i \; \frac{a_i(t)\, b_i(t)\, c_i(t)}{P_i^2(t)} \, e^{j[\alpha_i(t) + \beta]} \qquad (3\ 1\ 24)$$

Let $\tilde{f}(t)$ be the complex envelope of the transmitted signal, then the complex envelope of the echo signal from the ith scatterer is given by

$$\tilde{S}_{ri}(t) = \tilde{V}_i\left(t - \frac{\lambda_i(t)}{2}\right) f(t - \lambda_i(t))\; e^{j\omega_c \lambda_i(t) + j\beta} \qquad (3\ 1\ 2_{\prime})$$

where ω_c is the carrier frequency and $\lambda_i(t)$ is the round trip delay.

$\lambda_i(t)$ as given in (2 2 36) is

$$\lambda_i(t) = \lambda_i - 2\, \frac{v_i t}{c} \qquad (3\ 1\ 26)$$

where v_i is positive if the scatterers are moving towards the radar.

Substituting

$$\tilde{x}_i(t) \triangleq \tilde{V}_i\left(t - \frac{\lambda_i(t)}{2}\right) e^{j\omega_c \lambda_i(t) + j\beta} \qquad (3\ 1\ 27)$$

eqn (3 1 25) reduces to

$$S_{ri}(t) = \tilde{x}_i(t)\, \tilde{f}(t - \lambda_i(t)) \qquad (3\ 1\ 28)$$

Assuming that random phase β is uniformly distributed in $(0 \quad 2\pi)$ it can be seen that the process $x_i(t)$ has ero mean

Assuming that there are K scatterers whose echoe arrive with the same delay λ the total received signal with the delay λ is

$$\tilde{S}_r(t \quad \lambda) \quad \sum_{i=1}^{K} \tilde{x}_i(t)\, \tilde{f}(t \quad \lambda_i(t)) \qquad (3\ 1\ 29)$$

where $\lambda_i(t) = \lambda_i \quad \frac{2\, v_i}{c}\, t$

and

$$\lambda_i \quad = \lambda \quad \forall \quad i$$

Then the autocorrelation of the received signal with the delay λ is

$$R_{\tilde{S}_r}(t_\alpha \quad t_\beta \quad \lambda) = E\{ S_r(t_\alpha \quad \lambda)\, \tilde{S}_r^*(t_\beta \quad \lambda)\}$$

$$= E\{\sum_{i=1}^{K} \sum_{j=1}^{K} x_i(t_\alpha)\, x_j^*(t_\beta)\, f(t_\alpha \quad \lambda_i(t_\alpha))\quad \tilde{f}^*(t_\beta \quad \lambda_i(t_\beta))\} \qquad (3\ 1\ 30)$$

From (3 1 30) the autocorrelation function of the process x with the delay λ is

$$R_x(t_\alpha \quad t_\beta \quad \lambda) - E\{\sum_{i\ 1}^{K} \sum_{j\ 1}^{K} x_i(t_\alpha)\, x_j^*(t_\beta)\} \qquad (3\ 1\ 31)$$

Assuming that the scatter rs reflect energy independently and since the $x_i(t)$s are zero mean random processes the above reduces to

$$R_{\tilde{x}}(t_\alpha \quad t_\beta \quad \lambda) - \sum_{i\ 1}^{K} E\{ x_i(t_\alpha)\, x_i^*(t_\beta)\} \qquad (3\ 1\ 32)$$

J ($x_i(t)$ is stationary

$$R_x(\tau \ \lambda) = \sum_{i=1}^{I} R_{xi}(\tau) \qquad (3\ 1\ 33)$$

Then the normali ed correlation function of the proce x i given by

$$\rho(\tau \ \lambda) = \frac{R_x(\tau \ \lambda)}{R_x(0 \ \lambda)} \qquad (3\ 1\ 34)$$

from (3 1 34) the scattering function (f λ) i giv n by

$$S(f \ \lambda) - \int_{\infty}^{\infty} g(\tau \ \lambda)\, e^{j2\pi f\tau}\, d\tau \qquad (3\ 1\ 35)$$

3 2 XAMPLE

In this section to illustrate the procedure des rib d in Section 3 1 with mor detail a pecial ca e of th model i con idered and an expression for th scattering function is evaluated for the same

Her it is assumed that the ellipsoids are fluctuating in size long the y axi but their size is constant along the x and z axe i e $b_i(t)$ is a random process and $a_i(t)$ $c_i(t)$ are constants

Let $a_i(t) = c_i(t) = a_i \quad \forall \quad i$

and $M_i = 1 \qquad (3\ 2\ 1)$

It is al o assumed that $\phi_{ti} - 0 \quad \forall \quad i$

and $\phi \; - 0$

i e the y axis of the t c s and the y axis of the l c s are parallel With these assumptions the transformation matrix $[T_1]$ is as given below

$$[T_1] = \begin{bmatrix} \cos\theta_{t1}(t) & 0 & \sin\theta_{t1}(t) \\ 0 & 1 & 0 \\ \sin\theta_{t1}(t) & 0 & \cos\theta_{t1}(t) \end{bmatrix} \qquad (3\ 2\ 2)$$

From (3 1 11)

$$\begin{bmatrix} \sin\theta_1(t)\ \cos\phi_1(t) \\ \sin\theta_1(t)\ \sin\phi_1(t) \\ \cos\theta_1(t) \end{bmatrix} \quad \begin{bmatrix} \cos\theta_{t1}(t) & 0 & \sin\theta_{t1}(t) \\ 0 & 1 & 0 \\ \sin\theta_{t1}(t) & 0 & \cos\theta_{t1}(t) \end{bmatrix} \begin{bmatrix} \\ \\ \end{bmatrix}$$

$$(3\ 2\ 3)$$

Hence

$$\begin{aligned} \sin\theta_1(t)\ \cos\phi_1(t) & \quad \sin(\theta_{t1}(t) + \theta) \\ \sin\theta_1(t)\ \sin\phi_1(t) &= 0 \\ \cos\theta_1(t) &= \cos(\theta_{t1}(t) + \theta) \end{aligned} \qquad (3\ 2\ 4)$$

From (3 1 16) and (3 2 4)

$$P_1(t) \quad a \qquad (3\ 2\ 5)$$

Then the coordinates of the specular point are

$$x_i = \frac{a_i^2}{P_i^2(t)} \sin\theta_i(t) \cos\phi_i(t) \qquad a_i \sin(\theta_{ti}(t) + \theta)$$

$$y_i = \frac{b_i^2}{P_i^2(t)} \sin\theta_i(t) \sin\phi_i(t) \qquad 0 \qquad (3\ 2\ 6)$$

$$z_i = \frac{c_i^2}{P_i^2(t)} \cos\theta_i \qquad a_i \cos(\theta_{ti}(t) + \theta)$$

From the above the distance from the centre of the l c s to the specular point is

$$R_i - \sqrt{a_i^2} \quad a_i \qquad (3\ 2\ 7)$$

From (3 2 7) it can be inferred that the fluctuations of the ellipsoids about y axis alone do not have any effect on R_i

Rewriting (3 1 19a)

$$\alpha_i(t) \quad \frac{4\pi}{\lambda_c} [P_i(t) + x_{ti}(t) \sin\theta \cos\phi +$$

$$y_{ti}(t) \sin\theta \sin\phi + z_{ti}(t) \cos\theta] \qquad (3\ 2\ 8)$$

Substituting for $P_i(t)$ from (3 2 1) and (3 2 5)

$$\alpha_i(t) = \frac{4\pi}{\lambda_c} [a_i + x_{ti}(t) \sin\theta \cos\phi + z_{ti}(t) \cos\theta] \qquad (3\ 2\ 9)$$

In the above (x_{ti} y_{ti} z_{ti}) is the centre of the l c s This does not change with fluctuations in θ_{ti} Therefore the phases of the returns from the specular point are uneffected by the fluctuations in θ_{ti} x_{ti} z_{ti} can be written as

$$x_{ti}(t) = R_{ti}(t) \sin\theta_{li} \cos\phi_{li}$$

$$z_{ti}(t) = R_{ti}(t) \cos\theta_{li} \qquad (3\ 2\ 10)$$

where $R_{ti}(t)$ is the distance between the centres of l c s and t c s and is given by

$$R_{ti}(t) = \sqrt{x_{ti}^2(t) + y_{ti}^2(t) + z_{ti}^2(t)}$$

and θ_{ii} ϕ_{ii} are the aspect angles of the centre of the l c s in t c s

Substituting (3 2 10) in (3 2 9)

$$\alpha_i(t) = \frac{4\pi}{\lambda_c}[\, a_i + R_{ti}(t) \sin\theta_{ii} \cos\phi_{ii} \sin\theta + R_{ti}(t)$$

$$+ R_{ti}(t) \cos\theta_{ii} \cos\theta\,] \qquad (3\ 2\ 11)$$

Since the ellipsoids are assumed to have linear velocities along the line of sight

$$R_{ti}(t) = R_{ti}(0) - v_i t$$

Therefore

$$\tilde{x}_i(t) = \frac{G\lambda_c}{8\pi R_o^2} \quad q\, b_i(t \quad \frac{\lambda_i(t)}{2}) \exp[\quad j\frac{4\pi}{\lambda_c}\{a_i + R_{ti}(t \quad \frac{\lambda_i(t)}{2})$$

$$(\sin\theta \sin\theta_{ii} \cos\phi_{ii} + \cos\theta \cos\theta_{ii})\} - j\omega_c \lambda_i(t) + j\beta\,]$$

Assuming that $v_i << c$,

and substituting $c_1 = \sin\theta \sin\theta_{ii} \cos\phi_{ii} + \cos\theta \cos\theta_{ii}$

$$\hat{x}_i(t) = \frac{G\lambda_c}{8\pi R_o^2}\ b_i(t \quad \frac{\lambda_i(t)}{2}) \exp[\quad \frac{j4\pi}{\lambda_c}(a_i + (R_{ti}(0) + \frac{v_i\lambda_i}{2})c_1)$$

$$- j\omega_c\lambda_i + j\beta + \frac{j4\pi}{\lambda_c} c_1 v_i t + j\omega_c \frac{2v_i}{c} t] \qquad (3\ 2\ 12)$$

β is the random phase incurred in the reflection proce s A suming β to be uniform in (0 2π) it can be seen that the process $x_1(t)$ has zero mean

The autocorrelation of the process $x_1(t)$ is given by

$$R_{x1}(t_\alpha \quad t_\beta) - E\{x_1(t_\alpha)\ x_1^*(t_\beta)\}$$

$$= \frac{G^2\ \lambda_c^2}{8\pi R_o^2}\quad E\{b_1(t_\alpha \quad \frac{\lambda_1(t_\alpha)}{2})b_1^*(t_\beta \quad \frac{\lambda_1(t_\beta)}{2})$$

$$E\{\exp[\,j\,\frac{4\pi}{\lambda_c}\,c_1\quad v_1(t_\alpha \quad t_\beta) + j\omega_c\,\frac{2v_1}{c}\,(t_\alpha - t_\beta)]\} \qquad (3\ 2\ 13)$$

where $\hat{b}_1(t)$ is independent of the rest of the random quantities in the above equation

Assuming $b_1(t)$ to be stationary

$$E\{\hat{b}_1(t_\alpha \quad \frac{\lambda_1(t_\alpha)}{2})\ b_1^*(t_\beta - \frac{\lambda_1(t_\beta)}{2})\} = R_{b_1}(\tau) \qquad (3\ 2\ 14)$$

where $\tau = t_\alpha \quad t_\beta$

Sub tituting $c_2 = \dfrac{G^2\ \lambda_c^2}{8\pi R_o^2}$ and $\omega_{d_1} - \dfrac{2\,\omega_c\,\lambda_1}{c}(1+c_1)$ (3 2 15)

From the above (3 2 14) reduces to

$$R_{x1}(t_\alpha, t_\beta) \quad c_2 R_{b1}(\tau)\ E\{\exp[+\ j\ \omega_{d_1}(-\tau)]\} \qquad (3\ 2\ 16)$$

The scatterers can move towards or away from the radar along the line of sight with equal probability with their velocity distributions centred about $\pm\bar{\omega}_d$

Therefore from (2 2 42)

$$E[e^{j\omega_{di}(\tau)}] \quad \cos(\bar{\omega}_{di}\tau)\phi_{\omega_{di}}(\tau) \tag{3 2 17}$$

Therefore

$$R_{xi}(t_\alpha, t_\beta) = \quad c_2 R_{bi}(\tau) \ \phi_{\omega_{di}}(\tau) \cos(\bar{\omega}_{di}\tau) \tag{3 2 18}$$

From the above it can be seen that $x_i(t)$ is a stationary process i e

$$R_{\hat{x}i}(\tau) = \quad c_2 R_{bi}(\tau) \quad \phi\omega_{di}(\tau) \cos(\bar{\omega}_{di}\tau) \tag{3 2 19}$$

From (3 1 33)

$$R_x(\tau \quad \lambda) = \sum_{i=1}^{K} R_{\hat{x}i}(\tau) - \sum_{i-1}^{K} c_2 R_{bi}(\tau)\phi\omega_{di}(\tau) \cos(\bar{\omega}_{di}\tau) \tag{3 2 20}$$

The normalized correlation function of the process x is given by

$$g(\tau \quad \lambda) = \frac{R_{\hat{x}}(\tau \quad \lambda)}{R_x(0 \quad \lambda)} \tag{3 2 21}$$

From (3 2 20) and (3 2 21)

$$g(\tau \quad \lambda) = \sum_{i=1}^{K} R_{\hat{b}i}(\tau)\phi\omega_{di}(\tau) \cos(\bar{\omega}_{di}\tau) \tag{3 2 22}$$

and the scattering function is

$$S(f \ \lambda) - \int_{\infty}^{\infty} g(\tau \ \lambda) \, e^{j2\pi f\tau} \, d\tau \tag{3 2 23}$$

Thus the cattering function can be calculated for other models with a procedure similar to the one described above For the scattering function to exist the assumptions on the random nature of various parameters should be made appropriately

CHAPTER IV

CONCLUSIONS

In this chapter the r sults obtained in Chapter 2 and Chapter 3 are summarised and some suggestions for urther work in this area are made

This thesi has attempted to provide a statistical characterization of clutter in terms of the catteiing function for two models of clutter returns Chapter 2 con tains the first model in which the scatterers are modelled as random rotating dipoles with an overall drift velocity and diff rential velocities An expression for the s attering function is derived by calculating the voltage reflection coefficients of an ensemble ot dipoles with a non homogeneous Poisson distribution An attempt has been made to gen rate a few scatt ring function which compare favourably with some of the reported results

Chapter 3 consi ts of a model in which th clutter targets are assumed to be a collection of ellip oidal catter rs with varying cross ections The ellipso d are as umed to fluctuate in size and also about their mean positions A an illu tration an expression for the scattering functi n is obtained for a relatively simple geometry and aspect angle fluctuations

Ґo evaluate thc adequacy of thc models for clutter considered ın thıs the ıs ıt ıs necessary to generate scatterıng functıons by extensıve sımulatıon for a wıde class of parametrıcally descrıbed probabılıty dıstrıbutıons and compare them wıth experımentally obtaıned sc tterın functıons on a more systematıc basis than has been attempted here

As clutter statıstıcs are well kno m to exhıbıt non stationary tatıstıcs ıt ıs also desirable to examıne whether these models can be used to generate uch tatıstıcs

BIBLIOGRAPHY

1 J L Wong I S Reed and Z A Kaprielian A Model for the Radar Echo from a Random Collection of Rotating Dipole Scatterers IEEE Trans Aerospace Vol AES 3 No 2 March 1967 pp 171 178

2 James W Wright et al On The Statistical Modelling of Radar Targets Army Missile Command Redstone Arsenal Alabama Nov 72 AD 753936 NTIS Report

3 H L Vantrees Optimum Signal Design and Processing for Reverberation limited Environments IEE Trans Military Electronics MIL 9 July 1965 pp 212 229

4 D C Childers and I S Reed A Model for the Power Spectrum of return d echoes from random collection of moving scatterers Univ of Southern California School of Engg Rept 102 Dec 1963

5 H L Vantrees Detection Estimation and Modulation Theory Part III John Wiley & Sons Inc 1971

6 Maurice W Long Radar reflectivity of land and sea Lexington Books 1975

7 J W Crispin and K E Siegel Methods of Radar Cross section Analysis Academic Press 1968

8 D L Kerr Propagation of Short Radio Waves M I T Rad Lab Series Vol 13 N Y McGraw Hill 1950

9 J L Lawson and G E Uhlenbeck Threshold Signals M I T Rad Lab Series Vol 24 N Y McGraw Hill 1950

10 Collins and Zucker Antenna Theory Part II McGraw-Hill 1969

11 Corson and Lorrain Introduction to Electromagnetic Fields and Waves W H Freeman and Co 1962

12 G T Ruck D E Barrick W D Stuart and C K Krichbaum Radar Cross section Handbook Plenum Press N Y 1970

13 J F Siegert On the fluctuation in signals returned by many independent scatterers M I T Rad Lab Cambridge Mass Rept 465 Nov 12 1943

14 E J Kelly and L C Lerner A mathematical model for the radar echo from a random collection of scatterers M I T Lincoln Lab Lexington Mass Rept 123 June 15 1956

15 M I Skolnik Introduction to Radar Systems McGraw-Hill Kogakusha Ltd 1926

16 A Papoulis Probability Random variables and stochastic processes McGraw Hill Kogakusha Ltd 1965

17 E Parzen Stochastic Processes Holden Day Inc 1962

18 S Karlin and H M Taylor A first course in stochastic processes Academic Press Inc 1975

19 F L Nathanson and J P Reilly Clutter statistics which affect radar performance analysis Supplement to the IEEE Trans on Aerospace and electronic systems Vol AES 3 No 6 Nov 1967

20 R L Mitchell Radar signal simulation Artech House Inc 1976

21 Stevenson A F Electromagnetic cattering by an ellipsoid in the third approximation J Appl Phys 24 1143 1953

22 J T Evans and T Hagfors Radar Astronomy M I T Lincoln Lab McGraw Hill 1968

23 David K Barton Radars Volume V Artech House 1975